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The Mathematician Felix Klein and His Lasting Impact

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Sommario: *Il gentile invito a scrivere un articolo per questa rivista in occasione del centenario della morte di Felix Klein (22 giugno 2025) mi offre l'opportunità di evidenziare alcuni dei punti più importanti della vita e dell'opera di questo matematico, organizzatore scientifico e riformatore dell'istruzione. Felix Klein (1849-1925) ha spinto numerosi ricercatori ad analizzare il suo lavoro e ad esaminare la sua influenza sullo sviluppo della matematica e delle sue applicazioni. Ciononostante, c'è ancora molto di nuovo da scoprire (si vedano i recenti libri Ciesielska et al. 2025; Tobies 2025). Allo stesso tempo, ci sono ancora da correggere alcune inesattezze ed errori, dato che nella letteratura e negli archivi online leggiamo ancora informazioni errate e affermazioni congetturali.*

Abstract: *The kind invitation to write an article for this journal on the occasion of the centenary of Felix Klein's death (22 June 2025) gives me the opportunity to highlight some of the most important points in the life and work of this mathematician, scientific organiser and educational reformer. Felix Klein (1849-1925) has prompted numerous researchers to analyse his work and examine his influence on the development of mathematics and its applications. Nevertheless, there is still much new to be discovered (see the recent books* CIESIELSKA *et al. 2025;* TOBIES *2025). At the same time, some inaccuracies and errors should be corrected, as we still read incorrect information and speculative statements in the written literature and online data.*

Felix Klein's Motto in Life

Felix Klein was a talent at an early stage. He was 16 years old when he graduated from secondary school in his home town of Düsseldorf. In his *Abitur* essay on the topic "The Toils of Life Alone Teach us to Value the Blessings of Life", ([1]) Klein already expressed himself extremely wisely. He concluded this German essay with the words: "The most blessed are not those who, born in the lap of luxury, have possessed and experienced everything desirable from early age but rather those who, by gradually toiling with the hard struggles of life, have climbed from one step to the next." Referring to Psalm 90:10, he added: "Indeed, if a life has been precious, it has been, as the Psalmist says, one of toil and trouble." (See TOBIES 2021, p. 23). This sounds like Felix Klein's motto in life. He knew that his ancestors had had to work hard to get to the top (his paternal grandfather was a blacksmith). So Klein hardly gave himself a break, but kept working until the very last moment.

Having been a full professor at the Bavarian University of Erlangen (1872-75), the École polytechnique/Technical University in Munich (1875-80), and the Saxon University of Leipzig (1880-86), Felix Klein joined the Prussian University of Göttingen in April of 1886. ([2]) His significant achievements in

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([1]) Source: Goethe, J.W. (1807). *Torquato Tasso* (Act V, Scene 1, Antonio speaking to Alfons).

([2]) In the German Empire, the individual states had cultural sovereignty, i.e. they had their own laws for education and science. Prussia was the state with the most universities, but was comparatively late in allowing women to enrol. It was Felix Klein who enforced the enrolment of women in mathematics and natural sciences in 1893 (initially with listener status, whereby each professor had to be asked individually for permission).

the fields of geometry, algebra, function theory etc. earned him international recognition. Richard Courant (1888-1972), who (after C. Carathéodory, 1913-17, and E. Hecke 1918-19) received Klein's chair in 1920, euphorically expressed: "His life was full of intellectual vigor and the will to act, both spurred by a brilliant imagination that was always contriving more and more new designs. He was entirely the sort of wise man and ruler described in Plato's *Republic*." (COURANT 1926, p. 211)

Recognized Results in Early Age

Before Klein was appointed professor in Erlangen at the age of 23, he had already attracted international attention. At the age of 20, he edited the second part of the book on Line Geometry ([3]) by his late university teacher Julius Plücker (†22 May 1868); Klein himself found the topic for his doctoral thesis from this area. ([4]) In March 1870, the French mathematician Gaston Darboux (1842-1917) approached Klein on the basis of his edition of Plücker's Line Geometry and asked him to collaborate on his newly founded *Bulletin des sciences mathématiques et astronomiques*. This was before Klein travelled to Paris in April 1870, where the Norwegian mathematician Sophus Lie (1842-1899) was already waiting for him. Klein worked on the *Bulletin* from 1870 to 1876, until he himself became the main editor of the journal *Mathematische Annalen* (*Math.Ann.* for short). ([5]) It is therefore not surprising that Klein's epoch-making article of 1871, in which he demonstrated the consistency of non-Euclidean geometries, also appeared in translated form in the *Bulletin* in 1871. ([6])

With his *Erlangen programme* (1872), Klein convincingly redefined geometry: geometric properties as invariants of transformation groups. ([7]) He continued to systematize mathematical theories by recognizing and explaining the interrelations between different disciplines. His visionary programmes concerned mathematics and its applications, but also history, philosophy, and didactics of mathematics. He was extraordinarily engaged, as his admirers would say, in raising awareness for the "eminent cultural significance of mathematics and its applications." ([8])

In 1892, the famous Austrian theoretical physicist Ludwig Boltzmann (1844-1906) extolled Klein's all-encompassing activity:

> [...] Klein's work encompasses almost all areas of mathematics. Especially noteworthy are his contributions to the following areas:
> 1. Algebra and its application to the theory of algebraic forms, number theory, geometry, the resolution of higher equations.

([3]) Plücker, J. *Neue Geometrie des Raumes, gegründet auf die Betrachtung der geraden Linie als Raumelement*. One volume edition (https://archive.org/details/neuegeoraum00plucrich/page/n5/mode/2up). Part I (pp. 1-226), Preface by A. Clebsch; Part II (pp. 226-374), ed. Felix Klein. Leipzig: B.G. Teubner, 1868/1869.

([4]) Klein completed his doctorate with top marks on 12 December 1868 at the Prussian University of Bonn under Rudolf Lipschitz (1832-1903), see TOBIES 2021, pp. 40-44. – Plücker did not supervise Klein's doctorate, what is written incorrectly here: https://www.mathgenealogy.org/id.php?id=7401, and also on the wikipedia pages (in 58 languages). The relatively long French wikipedia page based mainly on the book by Roberto Rodriguez del Rio et Martine Joulia (Trad.), *Une nouvelle conception de la géométrie : Félix Klein*, Barcelone, RBA Coleccionables, 2018. Unfortunately, this book contains numerous other errors.

([5]) Klein took over the editorship of *Math.Ann.* in 1876 together with the Leipzig mathematician Adolph Mayer (1839-1908), see TOBIES/ROWE 1990. – Klein remained in this position until 1924 and regularly found further contributors in order to promote as many new areas of mathematics as possible (TOBIES 2021, pp. 53-58).

([6]) Klein, Félix (1871). « Sur la géométrie dite non euclidienne. » *Bulletin* t. 2 (1871), p. 341-351 (Extrait des *Nachrichten von der Königl. Gesellschaft der Wissenschaften zu Göttingen*, année 1871, n.°17, 30 août).

([7]) *Vergleichende Betrachtungen über neuere geometrische Forschungen* (Erlangen: Verlag von Andreas Deichert, 1872 (48 pp.). – This was a research programme that every newly appointed professor in Erlangen had to publish at the time. The programme has been translated into Italian (1890), French (1891), English (1893, for a new English translation see ROWE 2025), Polish (1895), Russian (1896) and Hungarian (1897). It was also distributed in Spain and Argentina. In addition, there is a more recent Japanese translation in *Gendai sugaku no keifu* [Genealogy of Contemporary Mathematics], vol. 7. Tokyo: Kyoritsu Shuppan, 1970 (see the contributions by A. Ramirez and H. Kümmerle in CIESIELSKA et al. 2025).

([8]) [UAG] Math.-Nat. Fak. 25, Valentiner to Klein (Report, dated July 19, 1924).

2. General theory of functions, theory of elliptic, Abelian, θ-functions and of Riemann surfaces;
3. Theory of differential equations;
4. Foundations of geometry, curvature and other shape relations of curves and surfaces, also newer geometry and projectivity, the application of geometry to mechanics. ([9])

On this basis, Felix Klein was able to create a centre for mathematical and scientific research in Göttingen. He attracted more and more students from all over the world to Göttingen. New chairs and institutes were established, supported by funding from industry and the Prussian Ministry of Culture. After Klein succeeded in recruiting David Hilbert (1862-1943) to his side on 1 April 1895, he began joint research seminars on mathematics and its applications with him and eventually also with the other newly appointed colleagues in Göttingen.

Every lecture given in Klein's seminars is recorded in his seminar protocol books [Protokolle], making them a unique source. There are 29 volumes of minutes [Protokolle] from the years 1872 to 1912 preserved in Klein's estate in Göttingen; they allow a deep insight into his working methods. ([10])

Klein's Creation of a School for Mathematical Productivity Regardless of Nationality, Gender, Religion, and Area of Research

Shortly after Klein took up his first full professorship in Erlangen, he wrote to the above-mentioned French mathematician Gaston Darboux:

> Here in Erlangen I was at first very isolated, especially scientifically; my only interactions were with beginners. Since then, two older people have come here from Göttingen who work independently on geometry. I hope very much that this number increases next semester and that, in Erlangen, I will gradually succeed in establishing a school of geometric productivity such as that which had gotten off to such a great start in Göttingen while Clebsch was still there. ([11])

The aim of deliberately founding a scientific school of mathematical productivity was by no means common at the time. Alfred Clebsch (1839-1872), with whom Klein habilitated at the Prussian University of Göttingen in January 1871, is said to have been the first mathematician of the 19th century to create such a school in the field of algebraic geometry. ([12]) And it should be mentioned that Clebsch had close contact with the older representatives of the Italian school of algebraic geometry, from which Felix Klein soon benefited.

Creating a maths school meant consciously inspiring young talents to new achievements. Important mathematicians at Berlin University such as Ernst Eduard Kummer (1810-1893) and Leopold Kronecker (1823-1891) rejected this completely. Kummer is said to have likened the request for a dissertation topic to "a young man asking him to suggest a pretty young girl with whom he could fall in love". ([13]) Kronecker expressed in a letter: "We neither want nor need a school, because in our absolutely clear science every new discovery can render the previous school knowledge worthless. [...] We can therefore expect nothing beneficial from such joint work. On the contrary, such work can only hinder the progress of mathematics." ([14])

Felix Klein's working style – orientated towards Clebsch – corresponded to his own inner urge to influence others and help them to make progress. This is well illustrated by a statement in a letter from Klein to Sophus Lie, his collaborator in the early 1870s: "Until now, there has been no mathematics in Bavaria, and the southern German student seems to be a lazy thinker. That said, my rather

([9]) Quoted from TOBIES 2021, p. vii.

([10]) For the most recent analyses, see CIESIELSKA et al. 2025; TOBIES 2025.

([11]) [Paris] 64: Klein to Darboux, November 29, 1872, emphasis original.

([12]) See Shafarevich, Igor (1983). "Zum 150. Geburtstag von Alfred Clebsch." *Math.Ann.* 266, pp. 135-140.

([13]) See Tobies (Chapter 1) in CIESIELSKA et al. 2025.

([14]) See the German source of the complete letter, written by Kronecker to Georg Cantor (1845-1918), dated September 18, 1891, in *Jahresbericht der Deutschen Mathematiker-Vereinigung* (short: *Jahresbericht der DMV*) 1 (1891), pp. 23-24.

strong social drive – if this is understood as the desire to affect other people – will presumably be satisfied." ([15])

Already as a *Privatdozent* under Clebsch in Göttingen, Klein supervised the dissertation of a mathematician to completion: Josef Diekmann (1848-1905). While still in this position at Göttingen University, Klein also inspired Ferdinand Lindemann (1852-1939) – who later (1882) became famous for the first proof of the transcendence of π – and the Swiss Adolf Weiler (1851-1916) to produce results that led to their doctoral theses. Both completed their doctorates in Erlangen under Klein, because Clebsch had died unusually early in November 1872. ([16]) Although there were only very few students in Erlangen (there were never more than ten people in Klein's lectures), Klein succeeded in leading six of his students to a doctorate and one to a habilitation (as a *Privatdozent*). The latter, Aurel Voss (1845-1931), remained eternally grateful to the younger Felix Klein and later participated in many of his projects. Voss wrote of Klein's 'remarkable ability to recognise *the* point in the works of others that related specifically to his own ideas' and of Klein's 'talent for directing each of his students to the subject that best suited his particular talent and stage of development'. ([17])

Felix Klein, who travelled to Paris, Italy, Great Britain and the United States several times and was friends with many foreign colleagues, had a reputation for guiding young mathematicians to their own creativity. Sophus Lie, who knew early on of Klein's desire to guide, work with and help young talents, wrote to his Norwegian student Elling Holst (1849-1915) in 1874 recommending that he go to Klein: "Believe me, there is nothing for you in *Berlin* unless you were to be as lucky as I and meet another Klein there." ([18])

In total, Klein led more than 50 people to a doctorate, including 13 foreigners, among them two women, the Englishwoman Grace Chisholm, md. Young (1868-1944) and the American Mary F. Winston, md. Newson (1869-1959). ([19]) In addition, there are numerous people who have demonstrably completed their doctorates at other locations or abroad with the results achieved under Felix Klein. ([20]) A total of 164 people from abroad worked with him in his research seminars. ([21]) There were others who only attended the lectures.

Klein's courses have also been attended by people with doctorates from other places, who afterwards wrote to him enthusiastically to say that it was only with him that they really understood mathematics or that they had learnt to work mathematically. A notable example of this is Georg Pick (1859-1942), who came from a Jewish family, had completed his doctorate in Vienna and had already habilitated in Prague when he joined Felix Klein in Leipzig for two semesters. ([22]) Mellen Woodman Haskell (1863-1948), who studied with Klein the longest of Klein's American doctoral students and also provided the first English translation of Klein's *Erlangen programme*, said in a letter to Klein that he had 'finally learnt what it means to work. It is a hard thing to learn, but I hope it sticks with me,' and in his autobiography he noted once again that 'Klein expected hard work'. ([23])

([15]) [Oslo] A letter from Klein to Sophus Lie dated August 3, 1872. – For a discrepancy (in connection with Clebsch's *Connex* concept) that later emerged between Klein and Lie, see the analysis by Leslie KAY 2023.

([16]) See the list of Klein's doctoral students in KLEIN 1923 (GMA III), appendix, p. *11-13*.

([17]) VOSS 1919, p. 286.

([18]) Quoted from STUBHAUG 2002, p. 236.

([19]) See KLEIN 1923 (GMA III, Appendix, pp. *11-13*). – At that time, it was not yet possible to obtain a university entrance qualification at German girls' schools. Klein was therefore also committed to reforming girls' schools in Prussia (realised in 1908); his youngest daughter Elisabeth Klein, later widowed Staiger (1888-1968) was still able to benefit from this; she passed her teaching exams in mathematics, physics and English in 1913 and became the recognised headmistress of a girls' school – until the Nazis demoted her again in 1933. (She had studied for one semester at the famous Womens' College Bryn Mawr in the USA), see TOBIES 2008.

([20]) The latest findings on this are contained in TOBIES 2025.

([21]) See Chapter 16 (Moritz Firsching & Henning Heller) in CIESIELSKA et al. 2025.

([22]) See the correspondence between Klein and Pick, which was published with an introduction and comments in TOBIES 2023.

([23]) See Chapter 6 (Henning Heller) in CIESIESLKA et al. 2025.

As the number of students increased, Klein developed a system for preparing and following up the presentations in his research seminars in order to achieve the best possible effect for all participants and also for future students. The Japanese student Takuji Yoshiye (1874-1947), who took part in Klein's seminar in 1899/1900, left a report on the seminar preparation:

> German universities have something called a seminar, and it was said that the strength of German scholarship lay in this system. [...] Klein brought about ten problems, divided them among the applicants in groups of two or three, and gave precise instructions on when each problem was to be reported – setting specific dates for the first, second, and so on. Yoshiye, paired with a German student, was assigned the problem "Motion of an Individual Body in a Liquid." About a week before the deadline, Klein would invite the respective group to his home and ask what progress they had made. When they presented their findings, Klein would review them, point out any issues, and suggest revisions.
>
> In Yoshiye's case, when he showed Klein what he had written, Klein gave him specific instructions on how to revise it. However, his German partner had brought nothing at all. Klein was furious, saying it was unacceptable and questioned the purpose of attending the seminar if no work was done. He eventually told the partner to simply draw the diagrams during the presentation. Klein could be very strict in such situations. There was even an instance where Klein's assistant, a doctor, was assigned a problem and spoke on it for an hour, only for Klein to declare afterward that it was entirely unsatisfactory. ([24])

As part of the follow-up of the seminar content, everyone had to enter a handwritten summary of their presentation in one of the above-mentioned record books [Protokolle], which were available for all students / staff in the mathematical reading room. ([25])

Among the many people who were influenced, inspired and supported by Klein, the French student Georges Brunel (1856-1900) and the Danish student Poul Heegaard (1871-1948) deserve special mention. On the one hand, the example of Brunel is important in connection with the much-discussed correspondence between Klein and Henri Poincaré (1854-1912). On the other hand, the example of Heegaard sheds light on Klein's continuing conscious inspiration of young talents in comparison to French mathematicians.

From 1880/81, Brunel was the first Frenchman to join Klein in Leipzig for two semesters on the recommendation of Darboux. Klein felt obliged to lead Brunel to new results of his own. After Brunel had lectured three times at the seminar, Klein informed Darboux:

> I do have to write a few words to you about Brunel. He is remarkably receptive, and there is hardly anything that he has not read and really understood too. That said, the progress with his productivity has been slow; every attempt (and there have been many) to encourage him to work more boldly has so far failed. Only recently has he begun to study curvature radii, and I have of course supported his efforts, from which something publishable will hopefully emerge. This is not to say that he isn't very valuable to me personally. My interactions with him are friendly, but somewhat awkward. Despite his efforts, the German language still causes difficulties for him. ([26])

A short time later Klein reported progress: "Brunel had recently brought me a manuscript in which he had compiled formulae for curvature ratios of curves in n dimensions; I asked him to work this out for the *Annalen*." ([27]) Klein had found the right means and had pointed Brunel to Camille Jordan's 'Essai sur la géométrie à n dimensions' (*Bulletin de la Société Mathématiques de France*, t. III, 103-173). Brunel completed his first work on 3 June 1881, 'Sur les propriétés métriques des courbes dans un espace linéaire à n dimensions', published in the *Math.Ann.* 19 (1882) 37-55. Brunel received a professorship in Bordeaux as early as 1884. We can interpret Brunel's later contribution to the *Encyklopädie der mathematischen Wissenschaften mit Einschluss*

([24]) See Chapter 12 (Harald Kümmerle) in CIESIESLKA et al. 2025.

([25]) The Mathematical Reading Room, a working room for students with a reference library, was an invention of Felix Klein. The room was open every day – even on Sunday – from 7 in the morning to 10 at night.

([26]) [Paris] 72-73: Klein to Darboux, May 3, 1881. (Klein used his native German in his letters; and Darboux wrote to Klein in French).

([27]) [Paris] 74: Klein to Darboux, May 28,1881.

ihrer Anwendungen (6 Vols., Leipzig: B.G. Teubner, 1898-1935), *ENCYKLOPÄDIE for short, as a thank you to Klein.* ([28])

After Brunel had submitted his mentioned first paper worthy of publication, Klein asked him to contact Henri Poincaré that same month. Klein thought that Brunel could now explain his own work to Poincaré, as Poincaré was working in a field of research that Klein had begun during his time in Munich. However, Brunel's letters to Poincaré, whom he had not previously known personally, were written in an unusually nationalistic tone. ([29]) This may have influenced Poincaré to be reluctant to share his own concrete methods with Klein. Klein sought a co-operation with Poincaré, as he had with many mathematicians from different countries; however, the exchange with Poincaré developed into a kind of competition (see TOBIES 2021, pp. 267–285).

Heegaard came to Göttingen in the summer term of 1894 after having spent time in Paris. He reported that his study visit to Paris (his teacher Hieronymus Georg Zeuthen (1839-1920) had sent him there) had not inspired him, but that he was subsequently greatly supported by Felix Klein. Heegaard attended Klein's lecture courses (differential equations, elementary geometry), and he gave the first presentation in Klein's research seminar in the summer term of 1894, in which he discussed the work of Maxwell and Sylvester on spherical functions. ([30]) Klein also invited Heegaard to give lectures at meetings of the Göttingen Mathematical Society, and Klein finally led Heegaard to his topic for the doctoral thesis:

> Klein had me give two lectures in the "Mathematische Gesellschaft" with a summary of Zeuthen's work on enumerative geometry. He also discussed with me the idea that would later form the basis for my dissertation. Altogether, there was a scientific atmosphere which stimulated me very much – stronger than anything I have ever met again. ([31])

Heegard's doctoral thesis, which he defended in Copenhagen in 1898, turned out to be an important contribution to modern knot theory. Moritz EPPLE (1899, pp. 240-58) discerned Klein's 'intellectual hegemony' in this discipline, emphasising the Austrian Wilhelm Wirtinger (1865-1945), who had studied with Klein in 1889. Both Wirtinger and Heegaard made important contributions to Klein's *ENCYKLOPÄDIE.* ([32])

Klein also succeeded in creating a school in applied fields. Karl-Eugen KURRER (2018) speaks of a Klein school in the field of structural analysis; the Nobel Prize winner in physics Eugene P. Wigner (1902-1995) spoke of 'Klein's school' in the field of mechanics and relativity theory. ([33]) Reinhard Siegmund-Schultze was able to show that the Austrian Richard von Mises (1883-1953), as one of the authors of Klein's *ENCYKLOPÄDIE (Volume 4: Mechanics) and as head of the Berlin Institute for Applied Mathematics in the 1920s, ultimately saw himself as Klein's true heir in this field.* ([34])

Klein had a similar influence on the fields of the social history of mathematics and the didactics of mathematics. He led the first mathematicians in Göttingen to habilitation in these fields, and encour-

([28]) Brunel, G. (1899). "Bestimmte Integrale" [Definite integrals]. *ENCYKLOPÄDIE, vol. 2.1.1, pp. 135-188.*

([29]) Brunel wrote to Poincaré (a letter dated 22 June 1881): « [...] il ne faut pas croire pour cela que je me plaise beaucoup au milieu de ce peuple; plus j'apprends à le connaître et plus je le déteste. [...] Je dois dire cependant que relativement j'ai toujours trouvé que Mr Klein était le plus aimable et le plus obligeant. Enfin, quels qu'ils soient, il faut savoir profiter de ce qu'ils ont de bon; c'est toujours là ce que je me suis dit et c'est même pour cela que je suis en Allemagne. [...] Je me mets tout entier à votre service. Français, notre devoir est de combattre les Allemands par tous les moyens possibles, mais loyalement. » (POINCARÉ 1986, pp. 91-92). See also NABONNAND et al. 2024.

([30]) [Protokolle] vol. 12, pp. 1-4 (Heegaard's handwritten entry on his presentation "Erzeugung aller ganzen, rationalen räumlichen Kugelfunctionen durch Differentiation" [Generation of all integer, rational spatial spherical functions by differentiation], 2 May 1894).

([31]) See TOBIES 2021, p. 395.

([32]) For Wirtinger, see in particular Peter Ullrich's Chapter 8 in CIESIELSKA et al. 2025. – In 1907, together with Max Dehn (1878-1952), Heegaard completed the contribution on topology (then still called *Analysis Situs*) for the *ENCYKLOPÄDIE*, vol. 3.1.1, pp. 153-220.

([33]) See also TOBIES 2025, pp. 441-444; 527-532; for Klein's integration of Emmy Noether into this area, see TOLLMIEN 2023 for details.

([34]) See R. Siegmund-Schultze (Chapter 13) in CIESIELSKA et al. 2025; and SIEGMUND-SCHULTZE 2021.

aged others to work in these areas. ([35]) He was far ahead of his time in supporting all avenues of mathematics, its applications, and instruction. His commitment to establishing new lecture courses, professorships, institutes, and curricula went hand in hand with innovative approaches to sourcing for research and instruction. Klein's reform of mathematical instruction included all educational institutions from kindergarten onward. He became the first president of the International Commission on Mathematical Instruction at the Fourth International Congress of Mathematicians in Rome in 1908 – although he did not have time to attend in person. ([36])

In 1912, when Felix Klein's former students came up with the idea of honouring him on the occasion of his 40th anniversary as a professor, around 330 people from all over the world donated money, including two women (mathematicians from St. Petersburg). ([37]) This enabled the persuasion of the renowned impressionist painter Max Liebermann (1847-1935) to create the portrait of Felix Klein that now adorns the Mathematical Conference Room at Göttingen University. Ten Italians were among the donors.

Felix Klein and Italian Mathematicians

Umberto Bottazzini has written at length about Bernhard Riemann's (1826-1866) influence on Italian mathematicians. Together with Jeremy Gray, he analyzed the development of complex function theory (Bottazzini/Gray 2013). Bottazzini and other Italian historians of mathematics have done much to reveal the close connections that Klein had with the Italian algebraic-geometric school of thought. Of particular interest are two collections of articles, one on mathematicians in Bologna (Coen 2012) and the other on the work of Corrado Segre (1863-1924) in Turin (Torino) (Casnati et al. 2016). Several editions of correspondence, moreover, document the fact that Klein maintained a close relationship with mathematicians in Italy (see Cremona 1992-99, Israel 2017; Menghini 1993; Luciano/Roero 2012). I was personally able to examine a collection of Klein's letters held in Pisa [Pisa].

Felix Klein's university professors Plücker and Clebsch already had good contacts with Italian mathematicians, ([38]) and Klein followed in their footsteps. As early as December 30, 1868, Klein had sent a copy of his doctoral thesis to Luigi Cremona (1830-1903):

> The study is concerned with an aspect of the theory of second-degree complexes. After his trip to Northern Italy this last fall, the now-deceased Plücker expressed more than once that you are the only person who fully understood him. I thus find it of the utmost importance to subject my first academic work to your esteemed judgement, all the more so because I achieved a result that deviates from that which lies at the basis of Battaglini's study of second-degree complexes. ([39])

Before Klein traveled to Italy for the first time, he had also informed Cremona about his edition of Plücker's *Liniengeometrie* and had asked him what he thought of Clebsch, so that he could use this in Clebsch's obituary. When Klein finally planned his first trip to Italy for August and September 1874, he wrote to Cremona on July 23 that he intended to travel to Rome via Switzerland with a friend, emphasizing that he would particularly like to meet him (Cremona), Giuseppe Battaglini (1826-1894); and Eugenio Beltrami (1835-1900). Klein met these and other Italian mathematicians (special mention should be made of Francesco Brioschi (1824-1897)), who were more than ten years older than him, and they appreciated him and his knowledge and later sent their own students to him so that they could continue their studies there. Cremona also accepted Klein's invitation to give a presentation in the

([35]) See Tobies 2025, pp. 471-73; 504-505.

([36]) See the latest findings on Klein's activities within the *International Commission on Mathematical Instruction* in Furinghetti/Giacardi 2022.

([37]) See the list of donors in Chapter 1 (Tobies) of Ciesielska et al. 2025. This list has already been included in the appendix of Tobies 2021. In the meantime, however, some misspelled names have been identified in the original printed greeting address, which has led to further discoveries.

([38]) See in particular the correspondence between Cremona and Clebsch, consisting of 28 letters, in Israel 2017, vol. I, pp. 347-393.

([39]) See here and below, the chapter "3.4 Trips to Italy" in Tobies 2021, pp. 153-158, and the original quotations in German in Tobies 2025, pp. 151-155.

mathematics section organised by Klein when the Society of German Natural Scientists and Physicians met in Munich in September 1877. ([40]) A few months later, Cremona wrote to Klein, who was 19 years his junior:

> Je me sens maintenant, plus que jamais, un de vos écoliers; car depuis plusieurs mois je n'ai fait autre chose qu'étudier vos travaux sur la transformation des formes binaires en elles-mêmes... ([41])

That time, Felix Klein undertook a second trip to Italy with his wife during the Easter vacation in 1878 (April 2nd to 25th). He had already informed Cremona of this in November 1877, adding that they could probably only travel to northern Italy. Klein and his wife Anna (née Hegel) (1851-1927), a granddaughter of the great philosopher G.W.F. Hegel (1770-1831), visited Pisa, Florence, Bologna and Venice. In Pisa, Klein finally managed to meet Enrico Betti (1823-1892), whom he had failed to meet in 1874. Altogether Klein could establish early good contacts with a number of important Italian mathematicians.

After Klein had visited Betti, Ulisse Dini (1845-1918) and others in Pisa in April 1878, Gregorio Ricci-Curbastro (1853-1925) came to Munich in the autumn on a scholarship. Ricci attended Klein's courses on number theory (1878/79) and algebraic equations (1879), and participated in the seminars. There he gave two lectures: on "Certaines équations du degré 2^n", and on a paper by Henry Stephen Smith (1826-1883), which had been published in French in an Italian journal: "Les courbes modulaires: Rapport sur un mémoire de M. Stephen Smith communiqué à la Académie Royale des Lyncées, 1877". ([42]) Klein had the latest literature on the subject analysed in the seminars and gave several presentations on his own findings. This increased Ricci's knowledge of the results of Riemann, Lipschitz and others. P. Speziali wrote: "Ricci greatly admired Klein, and his esteem was soon reciprocated [...]". ([43]) In 1899 Ricci had become a member of the German Mathematical Society (*Deutsche Mathematiker-Vereinigung*, short DMV), which published a speech by him in German in its 1902 annual report, in which he drew a historical arc from Euclid to non-Euclidean geometry. ([44]) When Ricci developed tensor analysis with his student Tullio Levi-Civita (1873-1941), Klein requested that these results be published in *Math.Ann.* ([45])

Luigi Bianchi (1856-1928) arrived in the summer semester of 1879 and stayed in Munich for three semesters. Unlike Ricci at the time, he had an excellent command of German and enjoyed an intensive collaboration with Klein, who first asked him to talk about his doctoral thesis, and he invited him to write an article about it for the journal *Math.Ann.* ([46]) Klein included Bianchi in his field of research and recommended topics for five further lectures. Thus, Bianchi made contributions to Klein's level theory [*Stufentheorie*] and spoke about the tetrahedral irrationality (on 29 May and 5 June 1880) and about the icosahedral irrationality (on 13 June, 4 and 11 July 1880). ([47]) In Klein's work "Über unendlich viele Normalformen des elliptischen Integrals erster Gattung" [On Infinitely Many Normal Forms of the Elliptic Integral of the First Kind], which was first presented on July 3, 1880 at the Academy of Sciences in Munich and was an effort to expand his level theory with the help of doubly periodic functions, Klein acknowledged Bianchi's results as follows: "At my request, Mr. Bianchi recently investigated the fifth level, and what I will communicate below are essentially results discovered by him." ([48]) In a follow-up article, Bianchi

([40]) See TOBIES 2021, pp. 206.

([41]) Cremona to Klein, Rome, 11 April 1878, published in ISRAEL 2017, vol. II.

([42]) [Protokolle] vol. 1.2, pp. 75-78 (Ricci's handwritten entry about his presentation, dated 28 July 1879).

([43]) Speziali, Pierre. "Ricci-Curbastro, Gregorio." *Dictionary of Scientific Biography*.

([44]) Ricci, Gregorio (1902). "Anfänge und Entwicklung der neueren Auffassungen der Grundlagen der Geometrie" (held in Padua on 5 November 1901). *Jahresbericht der DMV* 11, pp. 382-403.

([45]) Ricci-Curbastro, Gregorio; Levi-Civita, Tullio (1901). "Méthodes de Calcul différentiel absolu et leurs applications." *Math.Ann.* 54, pp. 125-201.

([46]) Bianchi, L. (1880). "Ueber die Flächen mit constanter negativer Krümmung." *Math.Ann.* 16 (1880), pp. 577-582.

([47]) Bianchi's seven lectures are recorded in [Protokolle] vol. 1.2, pp. 94-97; and vol. 2, pp. 6-10, 12-27.

([48]) KLEIN 1923 (GMA III), pp. 179-185, at p. 183.

expanded the proofs and, at the outset, thanked Klein "for his many suggestions and support in my work." ([49]) About the relationship between Bianchi's work and his own, Klein later explained:

> By treating, in the summer of 1880 (*Math.Ann.* 17), the elliptic curves that I would later call elliptic normal curves of the 3rd and 5th order by means of the *s* function, he overcame my reservations about using this resource and that of theta series. He [Bianchi] thus did his best to build a bridge from my work to the developments of Weierstrass's school, particularly to my friend [Ludwig] Kiepert's studies, which were written around the same time. ([50])

Conversely, Bianchi wrote to Klein: "I will never forget what you have done for me. And if I can be useful to you, remember that I am entirely at your disposal." ([51])

Bianchi soon obtained a professorship in Pisa, where Ricci became his colleague; both – it is well known – developed methods (*Bianchi identities* for the Riemann tensor), which play an important role in the general theory of relativity. Bianchi is famous for his work on differential geometry, about which he also wrote a comprehensive textbook *Lezioni di geometria differenziale* (Pisa 1894), which went through two German editions (1899, 1910) and was emphasised by Felix Klein. ([52])

When Klein moved to the University of Leipzig in October 1880, Giuseppe Veronese (1854-1917) came on the recommendation of the Swiss mathematician Wilhelm Fiedler (1832-1912) and Cremona; Klein supported Veronese's geometric work. ([53]) Francesco Gerbaldi (1858-1934), a student of Enrico d'Ovidio (1842-1933), came from Turin for the summer semester of 1883, and Giacinto Morera (1856-1909) for two semesters from autumn 1883, with a letter of recommendation by Eugenio Beltrami. Gerbaldi and Morera took part in Klein's research seminars, which at the time were dedicated to the theory of elliptic functions. Klein promoted publications by both of them. ([54])

Gerbaldi, who became a member of the DMV in 1897, had already copied Klein's *Erlangen programme*, the aforementioned booklet from 1872, during his stay in Leipzig in 1883. ([55]) In the same year, 1883, the Greek mathematician Cyparissos Stéphanos (1857-1917), with the support of Henri Poincaré, attempted to publish a French translation of Klein's booklet in the *Acta Mathematica*, but failed. ([56]) Ultimately, Corrado Segre was the first to arrange for Klein's *Erlangen programme* to be translated and published. Before, it is worth noting that Segre had adopted Felix Klein's motto 'Line geometry is like the geometry of a $M_4^{(2)}$ in R_5', when he wrote his doctoral thesis in Turin in 1883. After the appearance of Segre's study, Klein published a revised version of his own dissertation in volume 23 of *Math.Ann.* (1884). He referred to Segre's work, and he also accepted two of Segre's articles (one coauthored by Gino Loria (1862-1954)) for publication in the same volume 23. ([57]) They remained in close contact, and in November 1889, Segre, who was now a professor in Turin, asked for permission to translate the *Erlangen programme* into Italian. ([58]) Klein gave his assent. Segre's student Gino Fano (1871-1952), who would subsequently study under Klein, did the translation, and Klein supplemented the text with commentary. ([59]) This initiative from Turin was followed by further translations into other languages.

([49]) Luigi Bianchi. "Ueber die Normalformen dritter und fünfter Stufe des elliptischen Integrals erster Gattung." *Math.Ann.* 17 (1880), pp. 234-262, quotation p. 234.

([50]) KLEIN 1923, p. 6. – Ludwig Kiepert (1846-1934), a student of Karl Weierstraß (1815-1897) and a friend of Klein.

([51]) [UBG] Cod. MS. F. Klein 8: 91 (Bianchi's letter to Klein dated August 14, 1880).

([52]) See KLEIN 1927, p. 148. – Bianchi also refers to Klein's results, books and articles in his textbook.

([53]) For the Italian mathematicians studied with Klein in Leipzig, see TOBIES 2021, pp. 246-248.

([54]) See TOBIES 2021, pp. 244-245.

([55]) See Erika Luciano's Chapter 5 in CIESIELSKA et al. 2025.

([56]) See Christine Phili's Chapter 9 in CIESIELSKA et al. 2025.

([57]) See TOBIES 2021, p. 42, and TERRACINI 1926.

([58]) [UBG] Cod. MS. F. Klein 9: 991 (a letter from Segre to Klein dated November 19, 1889).

([59]) F. Klein. "Considerazioni comparative intorno a ricerche geometriche recenti." (Trans. G. Fano). *Annali di matematica pura ed applicata* 17 (1890), pp. 307-343.

In her work, Erika Luciano highlighted that, between 1883 and 1924, scholars in the Turin section of the Italian School of Geometry drew heavily on Felix Klein's ideas from Göttingen to advance research, teaching, and the organisation of mathematical life. ([60]) LUCIANO/ROERO (2016) stressed, that Klein was a "reference interlocutor" in particular for Corrado Segre, Gino Fano, Federigo Enriques and Guido Castelnuovo. It is remarkable what Segre wrote to Castelnuovo after visiting Felix Klein in Göttingen:

> No one who hasn't been here can imagine what breed of man Klein is, and what kind of organisation he was able, with a skill that no one else possesses, to impose on mathematical studies in this university. ([61])

After studying with Klein in Göttingen for two semesters (1893/94, 1894), Gino Fano wrote an article expressing his enthusiasm for Klein's teaching ([62]) and even became a serious candidate for Klein, who recommended him for a professorship in Göttingen in 1899 (TOBIES 2021, p. 447). Although Fano ultimately declined Klein's offer of an extraordinary professorship in geometry at the University of Göttingen, Klein had recommended him to the Prussian Ministry of Culture in Berlin. It was the extraordinary professorship that Arthur Schönflies (1853-1928) had held since 1893; in 1899, he had been appointed to a full professorship in Königsberg. On 6 February 1899, Klein wrote to Ludwig Elster (1856-1935), then a university councillor in the Prussian Ministry of Culture, that the person who was to be appointed as Schönflies' successor in Göttingen should

1. must be a man of science,
2. that he represents the geometric direction within mathematics and, if possible, is familiar with descriptive geometry,
3. that he has teaching talent and experience.

Since they could not find a German candidate who met all three criteria, Klein went on to write:

> Then it occurred to me that, under these circumstances, it might be possible to bring in a foreign candidate. I can now name someone who seems to offer the best guarantees in all three respects and who, when he was here in 1893-94, found his way into our German institutions and our language with remarkable ease. This is Dr Fano, *Privatdozent* at the University of Rome. ([63])

In this letter, Klein also asked whether it was possible in principle to appoint a foreign mathematician. Elster quickly replied in the affirmative. Thus, Klein, managing director of the mathematics and physics seminar at the University of Göttingen, sent the official appointment proposal to the ministry on 11 February 1899, with Gino Fano at the top of the list. ([64]) Since Fano had declined in the meantime, ([65]) Friedrich Schilling (1868-1950) was ultimately appointed. However, Schilling, one of Klein's students, was not rated as highly as Fano with regard to point 1. ([66]) Fano was appointed professor in Messina in 1899 and in Turin in 1901.

Klein maintained his excellent contacts with the mathematicians in Turin. It was here that he received his first honorary doctorate in 1880 (out of a

([60]) See the mentioned Chapter by E. Luciano in CIESIELSKA et al. 2025; and also LUCIANO/ROERO 2012.

([61]) CASNATI et al. 2016, p. 11 (Segre to Castelnuovo, Göttingen, 30 June 1891). – On the special role played by Segre's university courses in the heyday of the Italian school of algebraic geometry, see also CONTE/GIACARDI in CASNATI et al. 2016.

([62]) See FANO 1894, and also GIACARDI/LUCIANO/SCALAMBRO 2023. – A brief analysis of Klein's influence on the training of mathematics teachers in Italy can be found in GARIO 2006.

([63]) German original quotation: "Es ist mir also der Gedanke gekommen, ob es nicht möglich wäre, unter diesen Umständen einen ausländischen Kandidaten heranzuziehen. Ich weiß nun solchen zu nennen, der nach allen 3 Richtungen die vorzüglichsten Garantieen zu bieten scheint und der sich, als er im Jahre 1893-94 hier war, mit bemerkenswerther Leichtigkeit in unsere deutschen Einrichtungen und unsere Sprache hineingefunden hat. Das ist Dr. Fano, Privatdocent a.d. Universität Rom." [UBG] Cod. MS. F. Klein I C, Bl. 129 und 129r (Klein to Elster, letter draft, dated 6 February 1899).

([64]) [ZStA] Rep. 76 Va Sekt. 6 Tit. IV, Bd. XVII, Bl. 170-171r.

([65]) Klein had written to Gino Fano on 5 February 1899. Fano's negative reply, dated 10 February 1899, had not yet arrived before Klein wrote to Elster (11 February 1899). Fano thanked Klein, saying it was 'mere fantasy' and that he would prefer a chair in Italy. [UBG] Cod. MS. F. Klein 9: 4A, Brief v. 10.2.1899.

([66]) [ZStA] Rep. 76 Va Sekt. 6 Tit. IV, Bd. XVII, Bl. 170-171r. When Schilling was appointed to a full professorship in Danzig (now Gdansk, Poland) in 1904, the extraordinary professorship in Göttingen became the first full professorship in applied mathematics at a German university (Carl Runge).

total of ten). When Klein travelled to Italy in the interest of his major project, the *ENCYKLOPÄDIE,* he celebrated his 50th birthday (25 April 1899) in Turin. Klein was fêted by the mathematicians of this city, where his former doctoral student Grace Chisholm Young and her husband William Henry Young (1863-1942) took part. They just stayed for a period of study there and published their first works in Italian. ([67]) Klein recruited Italians as authors for the *ENCYKLOPÄDIE*; a total of 16 contributions were written by eleven Italian scientists, most of them for Vol. 3 (Geometry): Federigo Enriques, Gino Fano, Luigi Berzolari, Gino Loria, Guido Castelnuovo, Corrado Segre.

Ernesto Pascal (1865-1940) from Naples also deserves special attention. Pascal attended Klein's lecture courses on Abelian functions at the University of Göttingen in the winter term of 1888/89 and in the summer term of 1889. Klein encouraged him to publish, and as early as 1889 he submitted two articles by Ernesto Pascal on Abel's sigma functions to the *Göttinger Nachrichten*. ([68]) Pascal then wrote enthusiastically to Klein:

> J'ai ne pas des expressions pour vous dire avec quelle enthusiasme je pense à vous et à l'année passé avec vous à Göttingen; et votre portrait exposé dans une chambre sert à moi souvenir à chaque moment de l'année plus heureux de ma jeunesse. ([69])

Later, Ernesto Pascal also translated works by Felix Klein into Italian and arranged for Klein's membership of the Societá Reale di Napoli / Accademia delle Scienze Fisiche e Matematiche (1913). It is noteworthy that Pascal also formulated a prize task for the Academy in Naples, which was to deal with Klein's work on hyperelliptic and abelian functions (although no prize-worthy work was received). ([70])

Felix Klein joined the Circolo Matematico di Palermo, founded by Giovanni B. Guccia (1855-1914) in 1884, comparatively late. He became the 353rd member of the Circolo in 1905; in the same year Guccia planned to promote applied mathematics. Klein was elected for 1909-1914 as a member of an international board of the Circolo, also responsible for the journal *Rendiconti del Circolo Matematico di Palermo*. ([71]) This international 'steering committee' consisted of twenty members, including Klein and other Göttingen mathematicians at the time (Hilbert, E. Landau, Carathéodory). Those who donated for a Guccia medal in 1914 received a bronze copy of it. ([72]) Guccia was also one of the donors of the above-mentioned portrait of Felix Klein, which Liebermann painted in 1912.

Nationalism – Internationalism

Felix Klein's female doctoral student, the aforementioned Grace Chisholm, md. Young, concluded her obituary of Klein with the following sentences:

> The aim of his life was to knit together in unity of object and of effort the world of science, without distinction of nationality. By the irony of fate he is best known to many as one of the famous 93 who signed the manifesto, "It is true...," in October, 1914. As soon as the Armistice was signed I wrote to him urging him to let me know whether, as we supposed, his signature had been obtained by a ruse, such as I knew to have been the case with the astronomer Foerster. On December 7, 1918, Klein wrote me a circumstantial account of the matter. He first saw the document in the newspaper with his name printed below, he having telegraphed his consent when asked if he would sign an appeal to the intellectuals of the civilized world to maintain an objective attitude during the war. ([73])

([67]) See Chapter 10 (written by Elisabeth Mühlhausen) in CIESIELSKA et al. 2025.

([68]) Pascal, E. (1889). "Zur Theorie der ungeraden Abel'schen Sigmafunctionen dreier Argumente," *Nachrichten von der Kgl. Gesellschaft der Wissenschaften und der Georg-Augusts-Universität zu Göttingen*, pp. 416-423; and ibd. "Zur Theorie der geraden Sigmafunctionen dreier Argumente," pp. 547-553.

([69]) [UBG] Cod. MS. F. Klein 11: 164 (E. Pascal to Klein, 25 October 1889).

([70]) For Ernesto Pascal see in detail Maria Giulia Lugaresi's Chapter 7 in CIESIELSKA et al. 2025.

([71]) BONGIORNO/CURBERA 2018, pp. 198-199, 207-208; 258; 270; the Circolo reached 924 members by April 1914, more than any other mathematical society at the time (see pp. 205-207).

([72]) Ibid., p. 205; and [UBG] Cod. MS. F. Klein 115 (Collection of medals): Nr. 6.

([73]) Young, Grace Chisholm (1925). "Obituary. Professor Klein." *The Times*, Thursday, July 9.

As Bernhard vom BROCKE (1985) has already shown, older faculty members at German universities were unable to evade the "campaign to mobilize morale on the home front." More than three thousand university instructors in Germany (including Klein and Hilbert) felt compelled to demonstrate their loyalty to the state in declarations. There were several of such declarations, and Cordula TOLLMIEN (1993) has analyzed the extent to which Göttingen's professors participated in them. Felix Klein, the (only) elected representative of the University of Göttingen in the First Chamber of the Prussian Parliament from 1908 until the end of the German Empire in 1918, was better known than other mathematicians at the time. Thus he was the only mathematician asked (by telegram) to sign the appeal *Aufruf an die Kulturwelt* 'To the Civilised World' (in the words of G.Ch. Young: 'It is true...'), denying German war crimes. However, Tollmien's research makes it clear that Klein (and others, such as the astronomer Wilhelm Förster (1832-1921) and the physicist Max Planck (1858-1947)) had not seen the text; they naively assumed that the appeal was intended to calm the turbulence abroad. Nevertheless, Klein continues to be labelled a nationalist. ([74]) However, these rather strongly formulated categorisations do not stand up to a more differentiated analysis. Klein's stance against nationalism and chauvinism, which runs through his entire life, is substantiated with many original sources. ([75])

During his first stay in Berlin (1869/70) and also on many later occasions, Klein criticized nationalistically oriented Germans who made condescending remarks about foreign science and scientists. After studying in Paris with Sophus Lie in the spring and summer of 1870, Klein served for several weeks as a paramedic during the Franco-Prussian War. While doing so, he continued to work on mathematics. Even before the peace treaty was signed, moreover, Klein reestablished contact with Gaston Darboux. Whereas Darboux responded positively to Klein's overtures, other French (Camille Jordan (1838-1922), for instance) and German scientists maintained strong nationalistic positions. In 1870, Jordan himself annulled his membership in the Royal Society of Sciences (academy) in Göttingen. During the First World War, Felix Klein in Göttingen and Max Planck in Berlin ensured that the French members of the academies there would not be expelled. Émile Picard (1856-1941) left the Göttingen Royal Society (as Jordan once had) of his own volition. Picard was instrumental in banning German scholars from the Académie des Sciences in Paris. His main reason for doing so was to punish those who had signed the so-called appeal "To the Civilised World" ["Aufruf an die Kulturwelt"]. The appearance of Klein's signature on this much-discussed document, however, should not be considered in isolation.

From 1911 onwards, Felix Klein's wife Anna mentioned the "unrest in the world" and the "clamour of war" with great concern in her letters to her husband. ([76]) Felix Klein was aware of the rise of chauvinism, and in this context it is noteworthy that as early as 1908 he expressly opposed 'national chauvinism' in an official document to the Prussian Ministry of Culture and called for action to be taken against it:

> Particularly in our time, when national chauvinism is at its orgiastic peak, it must be close to the hearts of all truly educated people to counteract a movement that is increasingly intent on alienating even the best citizens of different nations from one another. It is as though the whole human spirit is vanishing along with the humanism that was once so rightly celebrated. [...] Intellectual exchange between modern cultures does not entail the elimination of national differences; rather, it leads to a clearer understanding of their true character and values, and this higher knowledge creates a friendly relationship among nations. (Quoted from TOBIES 2021, p. 511; German original in TOBIES 2025, p. 466)

In doing so, he was in agreement with the policy promoted by representatives in the Prussian Ministry of Culture. Shortly after the outbreak of the First World War in August of 1914, Klein submitted an article to the *Jahresbericht der DMV* in which he

([74]) Unfortunately David Rowe still propagated the view that Klein was an 'ardent nationalist'. See ROWE 2018a, p. 377.

([75]) See here and for the following TOBIES 2021, in particular sections 2.5.2; 2.7.1; 8.4; 9.1.1; 9.1.2.

([76]) [UBG] Cod. MS. F. Klein 10: 376 (Anna Klein to Felix Klein, September 29, 1911).

emphasized the international character of mathematics and mentioned the international projects that were in danger (KLEIN 1914). This included the work of the mentioned International Commission on Mathematical Instruction (ICMI), where Klein was elected president in absentia. In this context, it is worth mentioning that Castelnuovo wrote a letter of solidarity to Felix Klein (Rome, March 10, 1915), when disagreements arose between Henri Fehr (1870-1954), Secretary General of the ICMI, and Klein due to the "Aufruf an die Kulturwelt." ([77])

Little attention has been paid, until now, to the fact that Klein, in the midst of the war in 1916, emphatically supported a parliamentary memorandum in favor of expanding study abroad programmes. The aim of this initiative was to prevent future international conflicts. Furthermore, it should be stressed that, in 1917 and 1918, German journals published obituaries of the French mathematician Gaston Darboux, and these were written with Klein's consent. The obituary published by Aurel Voss in the *Jahresbericht der DMV* in 1918 was both longer and more detailed than that published by Hilbert the year before in Göttingen. ([78])

Despite what some authors have written, the sources do not support the image of Klein as a professor who was predominantly "loyal to the emperor" (and this initially surprised me as well). In several speeches given in the Prussian Parliament, Klein vehemently argued on behalf of improving education for all children (including those in kindergarten, elementary school, vocational school, and schools for girls). He was inspired, among other things, by the Perry movement, named after John Perry (1850-1920), and by books written by the mathematician and education reformer Benchara Branford (1867-1944) in Great Britain. Klein did not belong to any political party. He judged parties according to their results and thus even acknowledged, on occasion, the positive influence of the Social Democrats. ([79]) He did so as early as 1911, during the imperial era, and the Social Democratic Party was hardly "loyal" to the emperor, even though it finally consented to his wartime loans. It should be noted that, as a result of the approval of these loans, the Independent Social Democratic Party of Germany (*Unabhängige Sozialdemokratische Partei Deutschlands* = USPD) splintered off from the Social Democratic Party and that this new party was joined by Emmy Noether (1882-1935) and the philosopher Leonard Nelson (1882-1927), whose careers Klein helped to promote.

After the end of the First World War, Klein immediately offered his support to the new Prussian Ministry of Culture, which was led by Social Democrats. ([80]) As the mathematician Kurt Otto Friedrichs (1901-1982) reported, Klein deeply regretted the assassination of the Jewish Foreign Minister Walther Rathenau (1867-1922), who had become a member of the newly founded left-liberal German Democratic Party in 1918, and Klein feared for the prospects of the young republic. ([81]) Klein also rejected the nationalism that flared up after the war, as is clear from his autobiographical sketch published in 1923: "The few foreigners who worked with us in Göttingen at that time seemed to be a stimulating force in general; then there was no trace of the nationalistic antagonism that is now so prevalent in the public sphere." ([82])

Together with former students and colleagues from abroad, Klein helped to restore international relations after the war. To this end, he renewed contacts with scholars around the world and he made sure that corresponding members were elected to join the Göttingen Academy. The latter included Guido Castelnuovo in 1923, and Luigi Bianchi in 1924. ([83])

([77]) See FURINGHETTI/GIACARDI 2022, pp. 149-150, and TOBIES 2025, pp. 535-537.

([78]) See *Jahresbericht der DMV* 27 (1918) Abt. 1, pp. 196-217. In 1918, August Gutzmer (1860-1924), the then editor of the DMV's annual report [*Jahresbericht*] and one of Klein's close collaborators, published a profile of Klein in the middle of Voss's article for Darboux (between p. 216 and p. 217); Klein was honorary chairman of the DMV in 1918/19. – Hilbert, D. (1917). "Gaston Darboux." *Göttinger Nachrichten. Geschäftliche Mitteilungen*: 71-75.

([79]) See TOBIES 2021, Section 8.3.4.1.

([80]) Ibid., Section 9.3.2.

([81]) See ROWE 2018a, pp. 376-377.

([82]) KLEIN 1923a, p. 15.

([83]) [AdW Göttingen] Pers. 20: 1088, on Castelnuovo; Pers. 20: 1105, on Bianchi. The two nominations are written in Klein's hand.

Klein's tireless work ethic, which he had already internalized as a teenager, remained with him until his dying day. Klein made reforms when there was something to reform, and he also offered resistance when he deemed it necessary. He passed on this trait (at least) to his daughter Elisabeth, who, in 1933, was demoted from her position as a school principal and had to transfer to another school, as a teacher, on account of her opposition to the regime. ([84])

Otto Blumenthal on Klein's Impact

The mathematician Otto Blumenthal (1876-1944), born into a Jewish family, gave a speech in memory of Felix Klein on behalf of the DMV, when a memorial plaque was placed at his birthplace (Düsseldorf, Jägerhofstraße 11) in 1927. In his speech, Blumenthal posed the rhetorical question: "What justification do we have [...] to honour a pure scientist, the representative of a science that embodies abstraction in its purest form, in such a conspicuous manner?" And Blumenthal answered it himself:

> I would not bestow this honour on a specialist scientist who pursues his science as an end in itself and only delights and delights a small circle of connoisseurs with his discoveries. Whoever shall live on in the memory of the wide world must have had an impact on that world. That is exactly what Klein did. With an inner urge and a clear view, he sought and found the points where his science is rooted in life and from where it can intervene in life in a beneficial and transformative way [...].
>
> The great scientist rises above the specialist in his ability to summarise, to comprehensively subordinate individual facts to general principles. Before Klein's eyes lay the complex fabric of modern mathematics as a unified picture, held together by his own geometric and descriptive method. It was a summary of the creative impact that in Klein's hands and in the hands of many of his students gained and continue to gain brilliant treasures. [...]
>
> Mathematics visibly intervenes in life at two points: in the mastery of nature and in the education of young people. The concrete expression of the mastery of nature is modern technology, engineering. Klein recognised and proclaimed: Mathematics, exact science and technology are one, they must consciously work together for the benefit of mankind (and by the way: not for a war, R.To.). [...] Klein's most popular creation, the reform of mathematics and science teaching in secondary schools, is naturally linked to his great idea of unity. Even young people should absorb the life value of maths, which is why the mathematical approaches and methods that are effective in today's knowledge of nature must be taught at school. It was an overturning thought, a step against a tradition that goes back many centuries. [...] (Blumenthal 1928: *2-3*)

Felix Klein had strongly supported Otto Blumenthal's career (who completed his doctorate in 1898 under David Hilbert – whose first ever doctoral student). Thanks to Klein – who at that time had already led more than 50 people to doctorate-worthy results and increasingly left this task to Hilbert – Blumenthal was appointed professor at the Technical University of Aachen (RWTH Aachen) in 1905 and in 1906 became one of the editors (alongside Klein, Hilbert and Klein's former student Walter Dyck (1856-1934)) of the journal *Math.Ann.* (see Tobies 2021, p. 57). In 1898, Blumenthal was still critical of Klein's commitment to the application (see Rowe 2018, p. 89), but his work in engineering education at RWTH Aachen University led him to adopt a different perspective. In 1910, Blumenthal expressed the conviction 'that the inner content and outer appreciation of mathematics would gain a great deal if our most capable people wanted to think their way into engineering mathematics and create in it'. ([85])

Felix Klein on the Concept of Mathematics and the Responsibility of Mathematicians

Klein's inaugural lecture at the University of Erlangen was a plan for mathematical education, which he presented to a wide audience. This speech, delivered on 7 December 1872, contains remarkable statements

([84]) For Elisabeth Staiger (née Klein), see Tobies 2008.

([85]) [StB Berlin] Otto Blumenthal to an unknown addressee, in a letter from 1910.

about his perspective on mathematics.([86]) The twenty-three-year-old Klein possessed a firm opinion about the essence and the role of mathematics:

> One should not believe that the essence of mathematics lies in the formula; the formula is only a precise designation for the thought connections involved. [...] But the time is gone when the formula played the sole sovereign role at the expense of the thoughts behind it, and in which one regarded a mathematical work as finished so long as the computations were accessible. Today it is different: we require an inner understanding of the ongoing development, and consider a mathematical result complete only when it can be regarded from beginning to end as self-evident.

In his speech, Klein philosophized about the place of mathematics within the system of the sciences and within society at large. He emphasized both the *formal educational value* of mathematics as well as the value of its applications, in which regard he especially underscored "the theoretical services performed by mathematics in the development of other sciences." He cited examples of this from theoretical physics: the theory of light, molecular theory, geometric optics, the theory of heat conduction, and potential theory – topics in which he had immersed himself as a *Privatdozent* in Göttingen. He was aware that mathematicians were not appreciated at the time for the practical applications of their field that were "somewhat removed from the academic outlook," among which he mentioned the predictive calculations of astronomers, the precision of geometric measurements, and the accomplishments of engineers. This *academic* outlook was somewhat dismissive of technical applications – an attitude that Klein would do much to change in later years.

Klein's following statement from his inaugural lecture is also worth considering for more recent approaches to describing social processes using mathematics:

> It [mathematics] is not responsible if the conclusions do not agree with reality, any more than it can take credit for it if this is the case. Both depend on the correctness or incorrectness of the premises, and establishing them is not a mathematical business.([87])

When Klein's later work on approximation mathematics carefully compared the results obtained with empirical data, we can remember this early realisation. It saved him from jumping to conclusions and led him, among other things, to the realisation that is known today as the Painlevé-Klein problem. (See TOBIES 2021, pp. 471-472).

Klein also spoke explicitly about the mathematician's responsibility. In a lecture course that he gave in the summer term of 1901, he expressed:

> I wish to quote [Georg] Cantor, who once said: "*The essence of science lies in its freedom*," that is, mathematics can deal with anything it desires, as long as it draws only correct conclusions from premises. While I theoretically accept Cantor's sentence, I add a practical restriction, which seems to me essential, namely that *everyone who has freedom also has responsibility*. I do not want, therefore, to plead for an absolute arbitrariness in the construction of mathematical ideas, but I do want to recommend to everyone that they keep in mind the whole of science.([88])

Felix Klein extended Georg Cantor's maxim, which in its original form pertained only to *mathematics*,([89]) to science as a whole.

Klein's sense of responsibility for mathematics *and* its broad range of application may very well have led to his inclusion, on June 7, 1923, into the Order *Pour le Mérite für Wissenschaften und Künste* along with Albert Einstein, the writer Gerhart Hauptmann, the sculptor Hugo Lederer, and the painter Max Liebermann.([90]) The Weimar Constitution of 1919 had banned all Orders, among them

([86]) This speech was not published during Klein's lifetime. It was only the mathematician Konrad Jacobs (1928-2015) from Erlangen who brought the 19-page speech from Klein's estate to light (JACOBS 1977). – There is an English translation by David E. ROWE (1985).

([87]) JACOBS 1977, Klein's inaugural speech, p. 10.

([88]) Quoted from the English translation of the third edition of Klein's lecture course, KLEIN 2016 [^{3}1928], p. 170.

([89]) See Georg Cantor, "Ueber unendliche, lineare Punktmannichfaltigkeiten, 5." *Math.Ann.* 21 (1883), pp. 545-591, at p. 564: "The *essence* of *mathematics* lies precisely in its *freedom*." This translation is from EWALD 1996, p. 896.

([90]) See Orden Pour le Mérite für Wissenschaften und Künste, *Die Mitglieder des Ordens: Zweiter Band, 1882-1952* (Berlin: Gebr. Mann, 1978), p. 310.

the *military* class of this Prussian Order of merit, the history of which dates back to 1667. The *peace* class of this Order, however, which had been introduced in 1842 at the suggestion of Alexander von Humboldt, became, since the 1920s, a self-supportive "loose association of eminent scholars and artists," and it still exists today. Before Klein, the following mathematicians had been made members of the peace class (when it was still a distinction conferred by the Prussian state): Gauss (1842), Jacobi (1842), Cauchy (1849), Poncelet (1863), Weierstrass (1875), Hermite (1878), G.G. Stokes (1879), Carl Neumann (1897), Luigi Cremona (1902), and Ludwig Sylow (1904).

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